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ABSTRACT

Programmed materials presenting twelve mathematical concepts were developed to investigate disjunctive concept learning relative to four teaching strategies involving characterization and exemplification (CEC, EC, CE, ECE). Prospective elementary teachers in an undergraduate mathematics course were randomly assigned to one of eight treatment programs after stratification according to mental ability. Achievement tests designed to test concept attainment at the levels of Knowledge and Comprehension (Level 1), Application (Level 2), and Analysis, Synthesis, and Evaluation (Level 3) yielded the following results: (1) upper ability groups did significantly better than low ability groups; and (2) exemplar moves were more effective than non-exemplar moves for Level 1 performance, but the CEC strategy was better than ECE for Levels 2 and 3. No meaningful interactions were found. (Author/JG)

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THE RELATIVE EFFECTIVENESS OF FOUR STRATEGIES FOR TEACHING DISJUNCTIVE CONCEPTS IN MATHEMATICS

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Several studies have been made of the relative effectiveness of different strategies for teaching concepts. Those in mathematics have provided a little information, but the majority of the findings have indicated no significant differences exist between the different strategies studied (Henderson and Rollins, 1967; Rector and Henderson, 1970). One possible reason for this is the fact that these studies have not gone beyond the point of studying strategies for teaching concepts, as compared to strategies for teaching different types of concepts. That is to say, they have neglected to take into account the different logical forms of concepts.

A classification scheme for concepts was proposed by Jerome Bruner and several of his coworkers in A Study of Thinking (Bruner, Goodnow, and Austin, 1956). They divided the class of concepts into the following three subsets: conjunctive concepts, disjunctive concepts, and relational concepts. These three types of concepts are determined by the manner in which their defining conditions are combined in order to decide what may serve as an exemplar of the concept. A conjunctive concept is one determined by the joint occurrence of the appropriate values of its defining attributes. It is characterized by the use of the logical term "and". A disjunctive concept is one which is noted by the occurrence of at least one of the appropriate values of its defining attributes. It is set off by the use of the term "or". The third type of concept, the relational concept, is one which is

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determined by an explicit relationship between the values of the defining attributes (Bruner, et. al., 1956). The first two categories in this taxonomy of concepts have led to a multitude of psychological studies concerning the relative difficulty of attainment of conjunctive and disjunctive concepts.

The majority of these studies indicate that disjunctive concepts are significantly harder to attain than conjunctive concepts for subjects at all age levels (Snow and Rabinovitch, 1969). This difficulty with the set of disjunctive concepts has been attributed to several different factors. They ranged from the inability of subjects to profit from the information contained in non-examples of disjunctive concepts to confusion on the subject's part between the logical meanings of the terms "and" and "or" (Bruner, et. al., 1956; Hunt and Hovland, 1960). A recent study of the ability of secondary school students to read mathematics showed that the same problem exists in mathematics classrooms (LeDuc, 1971). The remainder of this paper deals with a study concerning the relative effectiveness of four strategies for dealing with disjunctive concepts in mathematics.

THE FACTORS OF EXPERIMENTAL INTEREST

As a result of the analysis of several studies into the nature of strategies used in the classroom teaching of concepts, four strategies were selected for use in the study. They were as follows:

1. Characterization-Exemplification Strategy (CE). An instructional strategy consisting of four characterization moves followed by six exemplification moves.

2. Characterization-Exemplification-Characterization Strategy (CEC). An instructional strategy consisting of two characterization moves followed by six exemplification moves followed by an additional set of two characterization moves.

3. Exemplification-Characterization-Exemplification Strategy (ECE). An instructional strategy consisting of three exemplication moves followed by a set of four characterization moves followed by an additional set of three exemplification moves.

4. Exemplification-Characterization Strategy (EC). An instructional strategy consisting of six exemplification moves followed by a set of four characterization moves.

These four strategies are a result of several descriptive studies made concerning the nature of strategies of teaching (Smith, et. al., 1967; Ginther and Henderson, 1966; and Anderson, 1968) and the model Henderson proposed (Henderson, 1967; Henderson, 1970).

A second factor of experimental interest was the nature of the moves contained within the set of exemplification moves. In order to attain a disjunctive concept through a series of exemplification moves, the student must learn to profit from the information carried in non-example moves. This results from the differing amounts of information available in positive and negative exemplars of the concept being studied. To cite an example of a particular disjunctive concept, it is sufficient for the exemplar to satisfy at least one of the disjuncts in the list of defining attributes. For example, an algebraic expression can be classified as a monomial simply by being a constant. It is not necessary that it fulfill all of the disjuncts, or defining conditions, as it would have to in a conjunctive situation As a result of this, a member of a disjunctive concept's referent set may only satisfy one of the several defining conditions for the concept. Hence, to attain a disjunctive concept through a sequence of positive exemplification moves amounts to a difficult information processing task.

A non-example of a disjunctive concept, on the other hand, must interact with each of the required conditions listed in the defining statement for the concept. For example, an algebraic expression is not a monomial if and only if it is not a constant and it is not a variable raised to a positive integral power and it is not a product each of whose factors is one of the above. This results from DeMorgan's Law for the formation of the negation of a disjunction (Exner and Rosskopf, 1959). The resulting statement is a conjunctive statement each of whose component parts is a negation of one of the disjuncts in the original statement. Thus, a non-example of a disjunctive concept must fail to satisfy each of the required conditions simultaneously. This requires a conscious effort at noting the values of each of the relevant defining conditions on the part of the learner.

Hence, in theory, it appears that exemplification approaches using predominately non-example moves might be more efficacious than those consisting predominately of example moves. To test this hypothesis, a factor involving the kind of exemplification move approach was crossed with the type of strategy factor. The exemplification approach factor had the following two levels:

1. Non-example Approach (~E) The six exemplification moves in each of the strategies in this combination of the factors were divided into four non-example moves and two example moves.

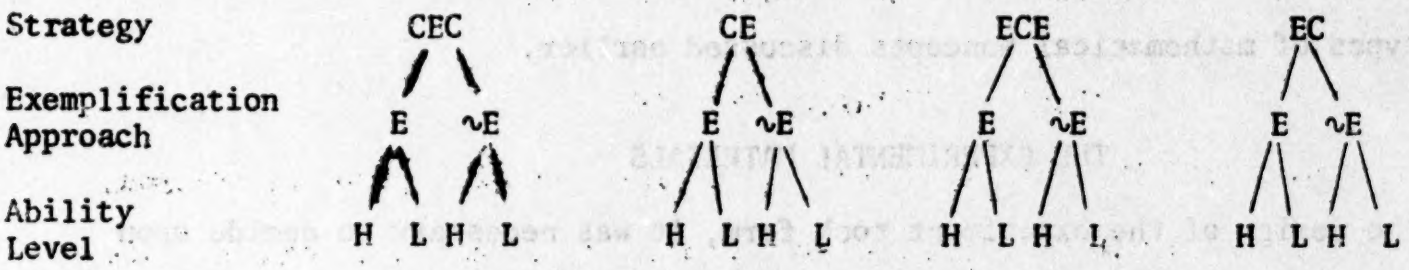
2. Example Approach (E) The six exemplification moves in each of the strategies in the combination of the factors were divided into two non-example moves and four example moves.

The insertion of this factor into the design of the study could have differing effects on students with different levels of intellectual ability. This led to the formation of a third factor of experimental interest. The factor of intellectual ability of the students involved was measured by the Henmon-Nelson Tests of Mental Maturity. The subjects were classified as being of either high or low ability as a result of their performance on the test.

Two other factors were built into the study, although they did not receive the status afforded to the foregoing factors. These were the recognition of the existence of two forms of disjunctive concepts and the different nature of algebraic and geometric concepts. In the first case, the set of disjunctive concepts can be split into two subsets depending on whether the word "or" is used in an inclusive or exclusive sense. In the first instance, the joint occurrence of the conditions is allowed, while in the latter it is not. To detect the effect that this might have on the attainment of disjunctive concepts, the concepts chosen for use in the study were split between the two classes.

In considering the nature of algebraic and geometric concepts, it seemed that they differed in the fact that the geometric concepts have an extra visual cue which the algebraic concepts do not. To test whether this had any influence on the rate of concept attainment, the concepts used in the study were split between algebraic and geometric concepts.

The resulting experimental design was a 4 X 2 X 2 completely crossed factorial design consisting of 16 cells. A pictorial model of the design is shown below:



LEVELS OF CONCEPT ATTAINMENT

In order to evaluate the relative efficacy of the four strategies and the two levels of exemplification approaches on the subjects of differing abilities, it was necessary to develop a test to measure the attainment of the different concepts studied. As Bloom and others have noted, the attainment of concepts takes place on several levels (Bloom, et. al., 1956). Hence, one factor might produce varied effects at different levels of learning. To test this out, the taxonomy of cognitive behaviors developed by Rector was used in the study. This taxonomy partitions the levels of cognitive activity into three main categories. It is similar to that proposed by Bloom, but more general in scope. The three levels are:

> Level I: Knowledge and Comprehension. This level of the taxonomy is concerned with the ability of the student to show evidence that he can remember factual material and translate communications effectively.
>
> Level II: Application. This level of the taxonomy is concerned with the ability of the student to discriminate between examples and non-examples of a given concept and to state what conditions allow or exclude a particular element as a member of the concept's referent set.
>
> Level III: Analysis, Synthesis, and Evaluation. This level of the taxonomy is concerned with the ability of the student to identify various subsets of the concept's referent set, to discriminate between the members of the referent set on the basis of their attributes, to produce methods for determining what may be contained in the referent set of a concept, and to evaluate methods of dealing with concepts themselves (Rector, 1970).

The selection of these levels provided general goals from which specific behavioral objectives were formed. The objectives were then used in the preparation of the experimental materials and an evaluation instrument. The test was constructed so

that it could be socred on the responses to the Level I items, the Level II items, the Level III items, and as a test in itself. It could also be graded to determine the subject's performance on the different types of disjunctive concepts and the different types of mathematical concepts discussed earlier.

THE EXPERIMENTAL MATERIALS

Once the design of the experiment took form, it was necessary to decide upon a medium of instruction which would allow strict control to be maintained over the information presented and the sequencing of the various types of moves. The decision was to use programed instruction. Programs were written for each of the eight forms of combinations of strategies and exemplification approaches. The only differences between these programs were the order of the moves, given in the description of the different sequences, and the changes necessary to satisfy the requirements of the exemplification approaches.

The twelve concepts selected for use in the study were contrived concepts involving familar mathematical concepts as their disjuncts. For example, a nint is a natural number which is a multiple of two or a multiple of five. A prifor is a natural number which id either prime or has a units digit of four.

The programs were constructed and tried out in three classes prior to their use in the experiment. In a like manner, the tests were tried out on each of these groups and refined for use in the experiment. The final test consisted of 72 questions, 24 at each of the three levels described earlier. It had a Kuder-Richardson reliability coefficient of 0.85. The subtests for Levels I, II, and III had reliability coefficients of 0.65, 0.64, and 0.69 respectively. Taking the shortness of these subtests into account, and the fact that the reliability of a test is a function of its length, the above coefficients were deemed satisfactory for experimental use of the test.

THE HYPOTHESES

The analysis of the nature of disjunctive concepts, the selection of the four strategies, the selection of the exemplifications approaches, and the differing nature of the students involved prompted the formulation of the following hypotheses for Level I responses, for Level II responses, for Level III responses, and for responses to the test as a whole:

1. There is no difference in the relative efficacy of the four strategies for teaching disjunctive concepts.

2. There is no difference in the example and non-example approaches in teaching disjunctive concepts.

3. There is no difference in the performance of the two intellectual ability levels in attaining disjunctive concepts.

4. There is no interaction between the types of strategies and the types of exemplification approaches in teaching disjunctive concepts.

5. There is no interaction between the types of strategies and the levels of intellectual abilities in teaching disjunctive concepts.

6. There is no interaction between the types of exemplification approaches and the levels of intellectual abilities in the teaching of disjunctive concepts.

7. There is no interaction between the three main factors in the teaching of disjunctive concepts.

In addition to these seven main hypotheses, the following two hypotheses were stated concerning responses to the entire tests:

1. There is no difference in the subjects' performance on the exclusive disjunctive concepts and their performance on the inclusive disjunctive concepts.

2. There is no difference in the subjects' performance on the algebraic disjunctive concepts and their performance on the geometric disjunctive concepts.

The first set of hypotheses was tested using a three-way analysis of variance and the second set was evaluated by use of a t-test for correlated samples.

THE SUBJECTS

The subjects consisted of 320 students selected from 363 students enrolled in two lecture sections of an undergraduate mathematics course for prospective elementary school teachers. The 363 subjects were ranked from one to three hundred

sixty-three on the basis of their performances on the Henmon-Nelson test. The top 160 students were divided into twenty groups of eight starting from the top. The students in each of these groups were then randomly assigned to the eight forms of the experimental materials for students from the high group. A similar procedure was followed in assigning the bottom 160 students to the treatment groups. The students were allowed to complete the programs during class time. This was followed by the test on the fifth day following the completion of the programs. An analysis of the time required to complete the eight forms of the programed materials showed no significant differences between the groups with respect to this measure.

INTERPRETATION OF THE DATA

The data from the tests was interpreted by means of a three-way analysis of variance. Prior to running the analyses, the data was checked to see whether it satisfied the assumption of homogeneity of population variances. Cochran's C-statistic was used for this purpose. The assumption of equal population variances was not rejected in any of the cases.

With this check it was possible to proceed with the analysis of variance. The 0.05 level of significance was selected for acceptance or rejection of the hypotheses stated for each test. The first set of data evaluated was that which dealt with the responses to the Level I items--those dealing with the areas of knowledge and comprehension. Table I contains the results of this analysis.

---Table 1---

The results of this analysis indicate that we must reject the null hypotheses dealing with the effects of type of exemplification approach and the levels of intellectual ability. An examination of the marginal means for these two factors indicate that the exemplification approach using predominately example moves is more effective than one using a majority of non-example moves. The finding that students in the upper ability groups did better than those in the low ability group was not unexpected. The remainder of the set of null hypotheses as they apply to the Level I test were not rejected.

TABLE I

ANALYSIS OF VARIANCE AND MARGINAL MEANS
FOR RESPONSES TO THE LEVEL I TEST

Source of Variance	Sum of Squares	df	Mean Square	F-ratio
A: Strategy	41.80	3	13.93	0.963
B: Exemplification Approach	63.90	1	63.90	4.416°
C: Ability Level	187.57	1	187.57	12.963°°
A X B	49.41	3	16.47	1.138
A X C	12.49	3	4.16	0.287
B X C	28.21	1	28.21	1.949
A X B X C	53.78	3	17.92	1.239
Within (Error)	4398.64	304	14.46	
Total	4835.84	319		

Marginal Means:

1.	Strategy Levels:	CE - 14.724	CEC - 14.987
		EC - 14.925	ECE - 14.075
2.	Exemplification Approaches:	Example ----- 15.124 Non-example-- 14.231	
3.	Ability levels:	High - 15.443	Low - 13.912

° Significant at the 0.05 level.
°° Significant at the 0.001 level.

The next set of data to be examined was the responses to the items making up the Level II test--those dealing with the application level of cognitive behavior. Table II contains the results of this analysis.

TABLE II

ANALYSIS OF VARIANCE AND MARGINAL MEANS
FOR RESPONSES TO THE LEVEL II TEST

Source of Variance	Sum of Squares	df	Mean Square	F-ratio
A: Strategy	152.87	3	50.95	3.830°
B: Exemplification Approach	41.31	1	41.31	3.106
C: Ability Level	232.90	1	232.90	17.508°°
A X B	66.66	3	22.22	1.670
A X C	0.78	3	0.26	0.019
B X C	14.05	1	14.05	1.056
A X B X C	57.08	3	19.02	1.430
Within (Error)	4043.94	304	13.30	
Total	4609.61	319		

Marginal Means:

1.	Strategy Levels:	CE - 15.037	CEC - 16.012
		EC - 15.149	ECE - 14.062
2.	Exemplification Approaches:	Example ---------- 15.424	
		Non-example ------ 14.706	
3.	Ability Levels:	High - 15.918	
		Low - 14.212	

° Significant at the 0.05 level.
°° Significant at the 0.001 level.

The results of this analysis indicate that the hypotheses dealing with the type of strategy and the different intellectual ability groups must be rejected at the 0.05 level of significance.

In order to ascertain the direction of the differences between the levels of the strategy factor, Duncan's New Multiple Range Test was used (Edwards, 1968). The results of that test are given in Table III. The 0.05 level was used to derive the least significant ranges for the test.

TABLE III

DUNCAN'S NEW MULTIPLE RANGE TEST APPLIED TO THE DIFFERENCES BETWEEN THE LEVEL II STRATEGY MEANS

	ECE	CE	EC	CEC
	14.062	15.037	15.149	16.012
ECE 14.062	-	0.975	1.087	1.950
CE 15.037	-	-	0.112	0.975
EC 15.149	-	-	-	0.863

Least Significant Ranges for $\alpha = 0.05$:

$R_2 = 1.128$ $R_3 = 1.188$ $R_4 = 1.228$

Any pair of means whose columns are not underlined by the same line differ significantly. Hence, the only statistically significant difference is between the ECE and the CEC strategies. A look at their means shows that the CEC strategy is better in helping students attain disjunctive concepts at the application level of behavior.

As in the analysis of the Level I responses, the students in the higher intellectural ability group did better than those in the lower intellectual ability group. This finding tends to support the manner in which the students were classified.

The next set of data to be analyzed was that which resulted from the subjects' responses to the items on the Level III subtest. This portion of the examination measured the subjects' ability to function at the analysis, synthesis, and evaluatio levels of the modified taxonomy. The results of this analysis are given in Table I These results, like those for Level II, indicate that we must reject the null hypotheses positing no significant differences exist between the levels of the strategy factor and between the levels of intellectual ability. Again the students in the high ability group achieved better scores on the test than those in the low ability group.

TABLE IV

ANALYSIS OF VARIANCE AND MARGINAL MEANS
FOR RESPONSES TO THE LEVEL III TEST

Source of Variance	Sum of Squares	df	Mean Square	F-ratio
A: Strategy	141.43	3	47.14	3.142°
B: Exemplification Approach	22.04	1	22.04	1.468
C: Abilith Level	495.01	1	495.01	32.990°°
A X B	16.10	3	5.36	0.357
A X C	3.42	3	1.14	0.076
B X C	8.46	1	8.46	0.564
A X B X C	126.07	3	42.02	2.800°
Within (Error)	4043.61	304	13.30	
Total	4609.61	319		

Marginal Means:

1. Strategy Levels: CE - 14.687 CEC - 13.362
   EC - 14.887 ECE - 13.362
2. Exemplification Approaches: Example----------14.756
   Non-example ---------- 14.231
3. Ability Levels: High - 15.737
   Low - 13.249

° Significant at the 0.05 level.
°° Significant at the 0.001 level.

The use of Duncan's New Multiple Range Test to determine the direction of the differences between the types of strategies found that the CE, EC, and CEC forms were all significantly better than the ECE form in helping the subjects attain the disjunctive concepts at this level. However, there were no significant differences between the CEC, EC, and CE marginal means. The results of this analysis are shown in Table V.

TABLE V

DUNCAN'S NEW MULTIPLE RANGE TEST APPLIED TO THE DIFFERENCES BETWEEN THE LEVEL III STRATEGY MEANS

		ECE	CE	EC	CEC
		13.362	14.687	14.887	15.037
ECE	13.362	-	1.325	1.525	1.675
CE	14.687		-	0.200	0.350
EC	14.887			-	0.150

Least Significant Ranges for $\alpha = 0.05$:

$R_2 = 1.200$ $R_3 = 1.263$ $R_4 = 1.306$

The results of the analysis of the scores on the Level III test also indicate that the hypothesis that there is no significant interaction between the three main factors must be rejected at the 0.05 level. This indicates that one of the two-way interactions is not the same for all levels of the remaining factor. An analysis of the possible combinations provided no interpretations which had meaningful interpretations affecting the teaching of disjunctive concepts (Dossey, 1971).

The final analysis of variance dealt with the scores from the three subtests combined. This set of scores was studied as it indicated the effectiveness of the different strategies and other factors as they acted over a broad range of educational objectives. The results of this analysis are given in Table VI.

TABLE VI

ANALYSIS OF VARIANCE AND MARGINAL MEANS
FOR RESPONSES TO THE TOTAL TEST

Source of Variance	Sum of Squares	df	Mean Square	F-ratio
A: Strategy	899.68	3	299.89	3.138°
B: Exemplification Approach	409.45	1	409.45	4.285°
C: Ability Level	2508.75	1	2508.75	26.257°°
A X B	199.14	3	66.38	0.694
A X C	31.09	3	10.36	0.108
B X C	171.17	1	171.17	1.791
A X B X C	547.10	3	182.36	1.908
Within (Error)	29045.78	304	95.54	
Total	33812.19	319		

Marginal Means:

1.	Strategy Levels:	CE - 44.200	CEC - 46.037
		EC - 44.962	ECE - 41.500
2.	Exemplification Approaches:	Example - - - - - - - - - - 45.306	
		Non-example - - - - - - - - 43.043	
3.	Ability Levels:	High - 46.974	
		Low - 41.375	

° Significant at the 0.05 level.
°° Significant at the 0.001 level.

The statistical analysis found significant differences between the component level means for each of the three main factors. No significant differences were found for any of the interaction factors, so the null hypotheses regarding them were not rejected.

The application of Duncan's New Multiple Range Test to the four strategy means indicated that the CEC strategy was significantly more effective in promoting the attainment of disjunctive concepts than the ECE strategy. The test also showed the

TABLE VII

DUNCAN'S NEW MULTIPLE RANGE TEST APPLIED TO THE DIFFERENCES BETWEEN THE TOTAL TEST STRATEGY MEANS

		ECE	CE	EC	CEC
		41.500	44.200	44.962	46.037
ECE	41.500	-	2.700	3.462	4.537
CE	44.200	-	-	0.762	1.837
EC	44.962	-	-	-	1.075

Least Significant Ranges for $\alpha = 0.05$:

$R_2 = 2.772$ $R_3 = 2.918$ $R_4 = 3.017$

EC strategy to be significantly more effective than the ECE strategy. The CE strategy is on the border line of being judged significantly better than the ECE strategy, but the difference of the means did not exceed the critical value of 2.772. The means underlined by the same line in Table VII dc not differ significantly.

An examination of the marginal means for the levels of the exemplification approach factor shows that the example approach was more effective than the non-example approach in promoting the learning of disjunctive concepts. An examination of the marginal means showed that the high ability students again did significantly better than the low ability students.

The above findings are a result of the three-way analysis of variance performed on the scores of the three subtests and the total test scores. Another pair of factors were balanced across the total test and were not analyzed by the analysis of variance. These were the factors concerned with the type of disjunctive concept and its mathematical nature. The relative difficulty of the levels of these factors was assayed by means of a t-test for correlated samples (Ferguson,1966)

The analysis of the difference scores between exclusive and inclusive items for the 320 subjects resulted in a t-value of 6.8721 with 319 degrees of freedom. This exceeds the tabled critical value for the t-test at the 0.001 level. Hence the hypothesis conjecturing that there was no difference in the nature of the two types of disjunctive concepts as reflected through the students' attainment of the as rejected. The results of the test indicate that students can perform better on exclusive concepts than they can on inclusive concepts.

The analysis of the difference scores between the geometric and algebraic items for the 320 subjects resulted in a t-value of 10.1287 with 319 degrees of freedom. This value also exceeded the tabled critical value for the t-test at the 0.001 level of significance. Hence the hypothesis stating that there was no significant difference between the performance of the students on the geometric and algebraic items is rejected. The results of the test indicate that students attain geometric disjunctive concepts easier than they attain algebraic disjunctive concepts.

INTERPRETATION OF THE FINDINGS

The results of the study of the relative efficacy of the four instructional strategies for teaching disjunctive concepts indicate that the logical form of a concept may have an affect on the success of a particular instructional strategy. This may be seen by comparing the results of the present study with the findings of a similar study that involved only conjunctive concepts (Rector and Henderson, 1970) The analysis of the data indicated that differences exist between strategies as one moves upward through the levels of cognitive behavior. These differences usually involved the CEC and ECE strategies.

These differences, at Levels II and III and on the total test, may be due to the formulation of the strategies and the logical nature of disjunctive concepts. It may be the case that the early introduction of the characterization moves in the CEC strategy indentifies and fixes the concept for the student. The following exemplification moves allow him to discriminate between examples and non-examples of the concept. The last two moves, both characterization, allow the student to

focus again on the characteristics and properties that help to set off the concept. Such an instructional strategy may prevent the initial confusion that may result from the nature of a disjunctive concept.

The ECE format, on the other hand, forces the student to infer the nature of a disjunctive concept from exemplification moves at the start of the strategy. It is only after three of the ten moves that the student first sees a characterization move. At this point, he may first start to focus on the relevant attributes and their values. Following four characterization moves, the strategy closes with three more exemplification moves. The arbitrary nature of a disjunctive concept may render the first three exemplification moves useless and the last three exemplification moves may not provide enough practice at discrimination between examples and non-examples of the concept for effective learning to take place. This is also suggested by the results of the EC strategy when compared to the ECE strategy. The EC format allows the student a longer exposure to exemplification moves. This may allow the student to derive his own mental model of the concept, which can be later adjusted in comparison to the information carried in the four characterization moves.

The fact that the means for the four instructional strategies were ordered in the same ascending manner of ECE, CE, EC, and CEC at each of the four levels of evaluation may also be important. Although there were no individual differences found among the CE, EC, and CEC means, the fact that the order was invariant under levels of evaluation may have some important interpretation in itself. It also appears that a strategy ending in a sequence of characterization moves may be a more effective strategy for handling disjunctive concepts than one ending in a sequence of characterization moves. Both of these conjectures should be examined by further research.

The type of exemplification approach used within the strategies also proved to make significant differences in the subjects' scores at Level I and on the total test. At both these levels, the approach using the four example moves and two non-example moves proved to be more effective than the one employing four non-example moves and two example moves. The difference at Level I, knowledge and

comprehension, may be due to the nature of questions at that level. This would no explain the overall difference though. Perhaps students at this level are not able to profit by knowing what a concept is not. Other studies in the use of examples and non-examples might include a factor concerning the logical nature of the concepts involved.

The difference in the subjects' scores on the exclusive and inclusive items may again be attributed to the logical difference in the two types of concepts. In the exclusive type there are no cases when an exemplar of the concept satisfies both of the conditions set forth in the disjuncts. In the inclusive case, some of the exemplars satisfy both of the disjuncts and in other cases satisfy one and not the other. This leads to a situation in which the inclusive concepts may not appear to be as well-defined as the exclusive concepts. It is here that the subjects might start to confuse the logical use of the terms "and" and "or". This seems a plausible source of the difference in the subjects' ability to handle the two types of disjunctive concepts.

The difference in the subjects' scores on the algebraic and geometric items, as indicated by the test items, may be attributed to the figures portraying the spatial relationships that accompany the textual information for the geometric concepts. Such conclusions should be drawn with caution however, for it is hard to compare the results on the basis of a lack of knowledge concerning the manner in which the information presented in a concept move is processed. Although an example move only gives an example, if it is algebraic, the example is a fact to be stored away. On the other hand, if the example concerns a geometric concept, then the example move may have an accompanying diagram. The effects of the different types of example information may create a situation in which an algebraic example move does not convey an equivalent amount of information as a geometric example move does. Such questions need to be explored further by carefully designed research.

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